Analysis of Supersonic-Hypersonic Flutter of Lifting Surfaces at Angle of Attack

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Several important features of supersonic-hypersonic flutter of lifting surfaces at angle of attack are highlighted in an exploratory study. Three simple analytical methods—a modified strip analysis incorporating shock expansion theory, modified Newtonian flow theory, and local flow piston theory—are employed in order to avoid the usual limitations of piston theory and Newtonian flow theory with regard to Mach number range, angle-of-attack range, and airfoil section. Illustrative flutter calculations have been made for two rectangular wings with diamond airfoils and with pitch and translational degrees of freedom. Results from the modified strip analysis and from the modified Newtonian flow theory are in reasonable agreement with limited available experimental data for Mach number 10, but results from the local flow piston theory are unconservative. The calculations show a typical degradation of flutter-speed index with increasing angle of attack, which at low angles of attack is most pronounced for thin sections. For some airfoil shapes, however, a forward location of the center of gravity may mitigate the degradation at low to moderate angles of attack and essentially postpone it until shock detachment conditions are approached. As angle of attack is increased, or Mach number is decreased toward the shock detachment condition, a sharp drop in flutter speed index is predicted by the methods that account for detachment. Thus, the vicinity of shock detachment is indicated to be a critical region for supersonic-hypersonic flutter.

Nomenclature

a = section pitch-axis location, units of b, positive aft of midchord
a_{cn} = section aerodynamic center location, units of b, positive aft of midchord
b = semichord
C_{m_α} = slope of section pitching-moment curve, $\partial/\partial\alpha_0$ (pitching moment about leading edge$/4q_\infty b^2$)
C_{N_α} = slope of section normal-force curve, $\partial/\partial\alpha_0(N/2q_\infty b)$
C_p = pressure coefficient, $(p - p_\infty)/q_\infty$
g = structural damping coefficient
k = reduced frequency, $\omega b/V_\infty$
M = Mach number
M_θ = section pitching moment about pitch axis
m_h = translating mass per unit span
m_θ = pitching mass per unit span
N = section normal force
q = dynamic pressure, $\rho V^2/2$
r_θ = section radius of gyration about pitch axis, units of b,

$$\left[\frac{\text{mass moment of inertia/unit span}}{m_\theta b^2}\right]^{1/2}$$

t/c = thickness ratio, maximum thickness/chord
V = speed
w = downwash or normal wash velocity
x_θ = distance from pitch axis to center of gravity of pitching mass, units of b, positive aft of elastic axis,

$$\frac{\text{static unbalance/unit span}}{m_\theta b}$$

α_0 = mean angle of attack
β = $(M^2 - 1)^{1/2}$
γ = ratio of specific heats
δ = flow deflection angle at airfoil surface
δ_w = wedge half angle ($\delta_w = \tan^{-1} t/c$ for symmetrical diamond airfoils, see Fig. 1)
μ = mass ratio, $m_\theta/(\pi\rho b^2)$
ξ = chordwise coordinate, units of b, positive aft of midchord
ρ = fluid density
σ = surface index as denoted in Fig. 1, $\sigma = 1,2,3,4$
ω = circular frequency
ω_h = natural frequency in normal translation
ω_θ = natural frequency in pitch

Subscripts‡

L = pertaining to local flow conditions
m = pertaining to infinitesimal motion
s = pertaining to stagnation conditions
∞ = pertaining to freestream

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Introduction

HIGH-LOAD factors associated with the supersonic maneuver requirements of missiles and manned aircraft, as well as the prospective operation of winged re-entry vehicles at high angles of attack in the supersonic hypersonic range, entail a need for accurate flutter prediction under these operating conditions. Flutter of vehicles of this type may be highly configuration dependent, but some limited analytical and experimental studies of simple lifting surfaces[1–4] have generally shown a degradation of supersonic flutter speed as angle of attack increases from zero. Thus, an aircraft might be aeroelastically stable at low angles of attack but flutter at higher angles. Clearly, the conventional procedure of making flutter calculations only for the zero angle-of-attack condition will not be sufficient for these supersonic-hypersonic vehicles.

A suitable flutter analysis technique must incorporate the effects of nonzero angle of attack and finite thickness, and should also be capable of including the aerodynamic influence of blunt leading edges which will probably appear on lifting re-entry and hypersonic cruise vehicles. The two most familiar aerodynamic theories that are used in flutter analysis for high-supersonic Mach numbers; namely, piston theory and Newtonian flow theory have serious limitations in application to wings at angle of attack. Piston theory is subject to the limitation that the product of Mach number and the angle of flow deviation from freestream direction must be less than 1.0. Hence, piston theory is not theoretically

‡ Dot over variable indicates differentiation with respect to time.

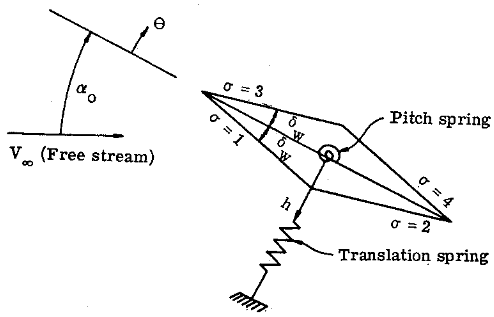


Fig. 1 Configuration used in illustrative computations.

valid even at zero angle of attack if the Mach number times maximum surface inclination exceeds unity, nor is it applicable for wings with blunt leading edges. Newtonian theory, on the other hand, is applicable for large surface inclinations and for blunt leading edges, but only for high Mach numbers. Some efforts to improve the theoretical description of oscillatory aerodynamic loads have involved relatively sophisticated studies of wave reflections, finite amplitude motions, and viscous effects,[5-9] but these have mostly been restricted to oscillating wedges, and have not been directed primarily toward the effects of angle of attack. More general discussions of hypersonic flutter may be found in Refs. 10 to 12; however, little information is given on flutter at angle of attack. A need clearly exists for an aerodynamic method that is applicable to a variety of wing sections over a wide range of supersonic-hypersonic speeds and at angles of attack from zero to at least the vicinity of the leading-edge shock detachment angle.

The objectives of this exploratory investigation are: 1) to examine three simple techniques—namely, a modified strip analysis,[13,14] modified Newtonian flow theory,[15,16] and local-flow piston theory[17]—for attaining these requirements; and 2) to use these techniques for some example flutter calculations which highlight some of the important features and problem areas of supersonic-hypersonic flutter at angle of attack as affected by variations in Mach number, airfoil thickness, structural damping, center-of-gravity position, and modal frequency ratio. Illustrative calculations have been made for two rectangular wings with diamond airfoils and with the pitch and normal-translation degrees of freedom shown in Fig. 1. The results have been compared with limited available experimental data.[1]

All of the present methods are based on the concepts of inviscid continuum flow; and hence, take no account of slip flow which may be significant at extremely high altitudes, nor of boundary layers and flow separation which are usually expected to be significant at high angles of attack. However, for sharp-edged wings at moderate to high angles of attack in supersonic flow and especially in hypersonic flow, the loading is predominantly determined by the very high pressure on the lower (particularly forward lower) surface. Therefore, the effects of separation of the low-pressure flow from the upper surface are not expected to be as profound as would be the case at lower Mach number.

Analysis

Methods

Modified strip analysis

References 13, 18, and 19 have shown that supersonic-hypersonic flutter of wings with sharp leading edges at zero mean angle of attack can be satisfactorily predicted by the modified strip analysis employing steady-state section lift-curve slopes and section aerodynamic centers given by shock expansion theory.[20] Any suitable alternate theory (e.g., characteristics or integral relations) can also be used to evaluate these required steady-state aerodynamic parameters, or measured values can be used.[21,22] For the latter, viscous effects and transonic flows are automatically incorporated, at least in terms of the steady flow.§ The modified strip analysis is not restricted to wings with sharp leading edges.

Application of this method for nonzero angle of attack is straightforward, and for the present configuration (Fig. 1), it is based on flow components parallel to and perpendicular to the wing chord (in which case normal force and normal force slope replace lift and lift curve slope in the formulation given in Refs. 13 and 14). Only minor geometric adjustments are required, and the resulting expressions for the oscillatory section normal force and pitching moment are

$$N = \pi\rho_\infty b^2(\ddot{h} + \dot{\theta}V_\infty \cos\alpha_0 - ba\ddot{\theta}) + (\rho_\infty b C_{N_\alpha} C V_\infty \cos\alpha_0)w$$

$$M_\theta = \pi\rho_\infty b^2[ba\ddot{h} + \dot{h}V_\infty \cos\alpha_0 + \theta V_\infty{}^2 \cos^2\alpha_0 - b^2(1/8 + a^2)\ddot{\theta}] - \pi\rho_\infty b^2 V_\infty \cos\alpha_0[1 - (a - a_{c_n})(C_{N_\alpha}/\pi)C]w$$

where the normalwash for the midplane surface is given by

$$w = \dot{h} + \theta V_\infty \cos\alpha_0 + b(C_{N_\alpha}/2\pi + a_{c_n} - a)\dot{\theta}$$

and C is in general a complex function of M and k (corresponding to the familiar Theodorsen circulation function for incompressible flow) which modifies the magnitudes and phase angles of the lift and moment vectors. For the high Mach numbers and very low reduced frequencies of this investigation, $C \approx 1 + i0$. For the present calculations, the required values of section normal force slope C_{N_α} and section aerodynamic center a_{c_n} are obtained from shock expansion theory[20]; and hence, the present applications are limited to angles of attack below and Mach numbers above "sonic-limit" values. This limit represents a condition very close to detachment for which the Mach number behind the leading-edge shock is unity and beyond which shock expansion theory is not valid.¶ This limitation of shock-expansion theory, however, does not apply to the modified strip analysis in general.

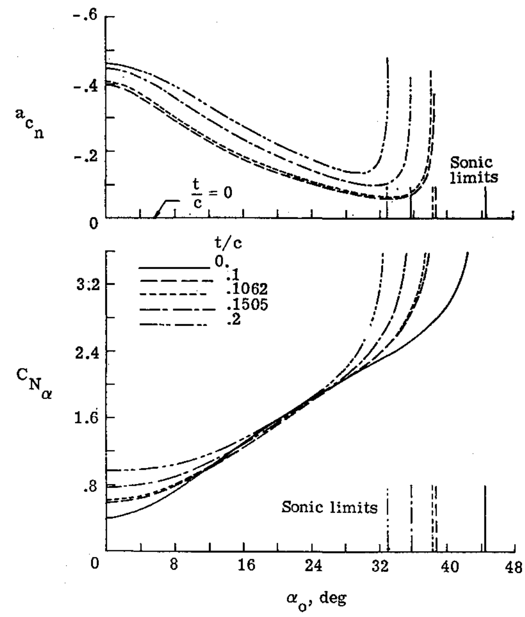
The C_{N_α} values for both $M = 2$ and $M = 10$ (Fig. 2) rise monotonically from $\alpha_0 = 0$ to asymptotes at the shock detachment angles. This asymptotic behavior occurs because calculated values of $\partial C_p/\partial\delta$ behind the leading-edge shock rise without limit as detachment is approached. For $M = 2$ the corresponding aerodynamic centers a_{c_n} are initially relatively invariant with increasing α_0 but move rapidly forward toward the quarterchord as detachment is approached and the forward lower-surface pressures predominate. In contrast, at $M = 10$ the aerodynamic centers move initially aft before the characteristic forward movement near detachment (Fig. 2a). For $\alpha_0 = 0$, similar trends of the aerodynamic parameters are shown in Fig. 2c as Mach number decreases toward detachment. These sharp changes near detachment

§ Measured steady-state global aerodynamic parameters have been used in Refs. 4 and 23 in quasi-steady supersonic flutter analysis for finite span wings at angle of attack. In that type of analysis, the aerodynamic damping terms, if included, must be evaluated by separate means, for example, by piston theory as in Refs. 4 and 23. Those investigations, however, gave results that are conflicting in some respects. For example, Ref. 4 found little effect of including pitch damping in the calculations; whereas, Ref. 23 indicated a significant effect. In the modified strip analysis employed in the present study, the corresponding damping terms are an integral part of the formulation.

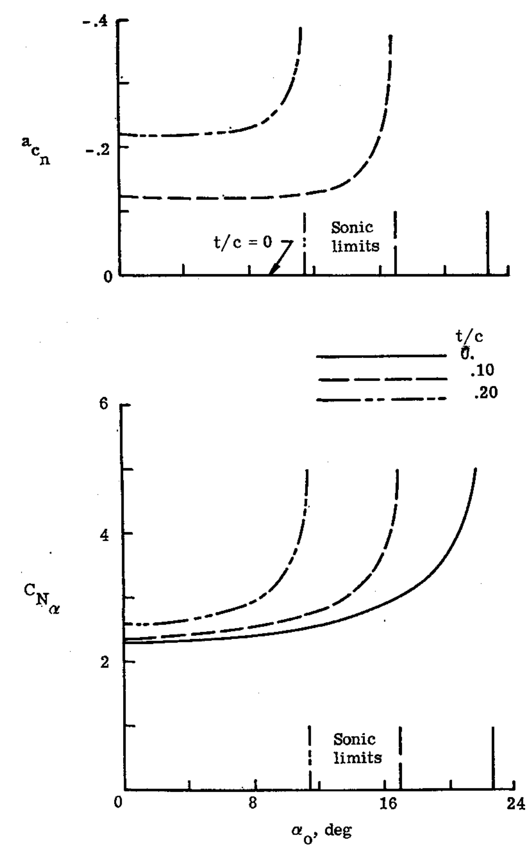
¶ Any increase in angle of attack or decrease in Mach number beyond this condition would result in subsonic flow in the region behind the shock. Hence, pressures over the forward wedge surface would in general no longer be constant, and the flow over the shoulder of the airfoil would no longer be represented by a simple Prandtl-Meyer expansion. Thus, for surfaces of finite extent, the shock-expansion theory is not valid beyond the sonic limit.

portend corresponding rapid variations in flutter boundaries as will be discussed in subsequent sections of the paper.

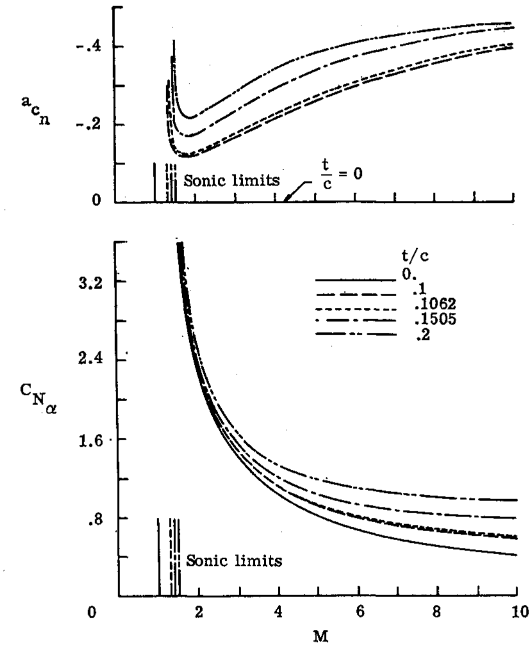
It is noted in passing that shock expansion theory was also used in Refs. 24 to 27 in combination with piston theory concepts in order to approximate the flow over a wing oscillating about a nonzero mean angle of attack. The flow was described in terms of fluid slabs oriented perpendicular to the wing mean chord plane and moving independently past an equivalent stationary wing having the same downwash distribution as the oscillating wing.



a) Variation with angle of attack at $M = 10$.



b) Variation with angle of attack at $M = 2$.



c) Variation with Mach number at $\alpha_0 = 0$.

Fig. 2 Normal force slope and aerodynamic center location from shock expansion theory.

Modified Newtonian theory

The modified Newtonian flow theory used herein assumes a point-function relationship between the local inclination angle and local pressure coefficient. Thus, the centrifugal force terms sometimes used in Newtonian flow theories are omitted, and the local pressure coefficient is given by $C_p = C_{p_s} \sin^2\delta$ for surfaces exposed to the free-stream and $C_p = 0$ for leeward surfaces, or

$$C_p = C_{p_s}(1 + v_m/V_\infty)^2 \sin^2(\alpha_0 \pm \delta_w + w_m/V_\infty)$$

or

$$C_{p_m} = 2C_{p_s}[v_m/V_\infty \sin^2(\alpha_0 \pm \delta_w) + (w_m/V_\infty) \sin(\alpha_0 \pm \delta_w) \cos(\alpha_0 \pm \delta_w)]$$

for exposed surfaces and

$$C_{p_m} = 0$$

for leeward surfaces, where for $\pm\delta_w$, the positive sign goes with $\sigma = 1$ and the negative sign with $\sigma = 2$ and 3 (see Fig. 1), and v_m and w_m are the velocity components of the motion parallel and normal to the freestream, respectively.

$$v_m = [\dot{h} + b(\xi - a)\dot{\theta}] \sin\alpha_0$$

$$w_m = [\dot{h} + b(\xi - a)\dot{\theta}] \cos\alpha_0 + V_\infty\theta$$

The two expressions used herein for stagnation pressure coefficient C_{p_s} are those obtained from hypersonic small disturbance theory,[16,28] $C_{p_s} = \gamma + 1$, and from normal shock relations,

$$C_{p_s} = \frac{2}{\gamma M_\infty^2}\left\{\left[\frac{2\gamma M_\infty^2 - (\gamma - 1)}{\gamma + 1}\right] \times \left[1 + \frac{(\gamma - 1)}{2}\left(\frac{2 + (\gamma - 1)M_\infty^2}{2\gamma M_\infty^2 - (\gamma - 1)}\right)\right]^{\gamma/\gamma - 1} - 1\right\}$$

It should be noted that the Newtonian-flow theory with $C_{p_s} = \gamma + 1$ is independent of Mach number.

Local flow piston theory

The local flow piston theory as used herein is based on piston theory type oscillatory perturbations from the steady flow given by shock expansion theory. Since the amplitudes of motion are assumed to be infinitesimal, only the first-order piston theory pressure term is used. The reference flow condition is considered to be the local mean flow at the surface rather than the freestream, so that

$$C_{pm} = (2/M_L)(q_L/q_\infty)w_m/V_L$$

where w_m is the motion induced velocity perturbation normal to the local mean surface position, positive outward. For a thick symmetrical diamond airfoil w_m is given by

$$\left.\frac{w_m}{V_L}\right|_\sigma = \mp\left\{\frac{\dot{h}}{V_L}\cos\delta_w + \theta - \frac{b\dot{\theta}}{V_L}\cos\delta_w[a + (-1)^\sigma \tan^2\delta_w - \xi(1+\tan^2\delta_w)]\right\}$$

where the upper sign is for the upper surfaces, ξ is a normalized chordwise variable, σ is the surface index (Fig. 1), and the pitch axis is assumed to be on the mean chordline.

A very similar local flow second-order theory has also been used which differs from the local flow piston theory only in having M_L replaced by β_L in the expression for motion related local pressure. (Compare with the second-order theory of Ref. 29.)

Since the local flow steady conditions are based on shock expansion theory for both cases, their application is also limited to angles of attack and Mach numbers for which shocks are attached.

Expressions for the flutter determinant elements obtained from all of the methods are given in Ref. 30.

Applications

The three analytical methods previously described have been employed in illustrative flutter and divergence calculations for two wings for which experimental flutter data at $M = 10$ and mean angles of attack up to about 10° were given in Ref. 1. These wings (referred to herein as wing 1062 and wing 1505, respectively) had 10.62% thick and 15.05% thick symmetrical diamond airfoils and rectangular planforms with panel aspect ratio 2.85. The pertinent physical properties are given in Table 1. The values of mass ratio μ given in the table are the experimental values from Ref. 1 for $\alpha_0 = 0$. Although the experimental values of μ varied somewhat with α_0, the values are so high that the flutter speed index is insensitive to minor changes in μ. Therefore, the values of μ listed in Table 1 were used in all the present calculations. A structural damping coefficient g of 0.02 was estimated in Ref. 1 to apply to the experimental investigation, and that value has been used herein also.

The wing models were relatively stiff, but the model support system provided pitching (θ) and translational (h) flexibilities. Since part of the support system translated with the model but did not rotate with it, the total moving masses m_h and m_θ given in Table 1 are not the same. The resulting flutter motion consisted essentially of rigid body translation and pitching with very little model deformation. Hence, displacement in the flutter mode was very nearly independent of spanwise location (i.e., approximately two-dimensional) even though the aerodynamic influence of finite span was present. However, the values of aspect ratio and experimental Mach number for these wings are high enough so that the latter effect should be small, and hence in the present calculations the wings are treated as two-dimensional (Fig. 1). The assumption of two-dimensionality is not essential to the use of the present methods and is made herein only for convenience. It is considered adequate for qualitative examination of some of the salient features of supersonic-hypersonic flutter at angle of attack and for demonstration of quantitative capability. It should be recognized, however, that in the lower supersonic range the present two-dimensional analysis would not be expected to accurately represent the three-dimensional wing of Ref. 1. Some comments on more accurate treatment of finite wings are given in the Appendix.

Table 1 Wing properties (Ref. 1)

Property	Wing 1062	Wing 1505
a	0.0	0.0
g	0.02	0.02
m_θ/m_h	0.6468	0.6496
r_θ^2	0.5289	0.5036
t/c	0.1062	0.1505
x_θ	0.2378	0.2321
ω_h/ω_θ	0.3966	0.3270
μ	36460.	39380.

Finally, it is noted that because of the limitations mentioned in the Introduction, ordinary piston theory (based on free-stream conditions rather than local-flow conditions) is not theoretically valid at any angle of attack for the wings studied herein at $M = 10$, the experimental value of Ref. 1.

For comparison with experiment, flutter calculations have been made by the three present methods for the models and flow conditions of Ref. 1 and for an extended range of angle of attack. In addition, a trend study employing the modified strip analysis has been made to show some of the salient effects of variations in Mach number M, angle-of-attack α_0, wing thickness ratio t/c, structural damping coefficient g, center-of-gravity position x_θ, and modal frequency ratio ω_h/ω_θ.

Results and Discussions

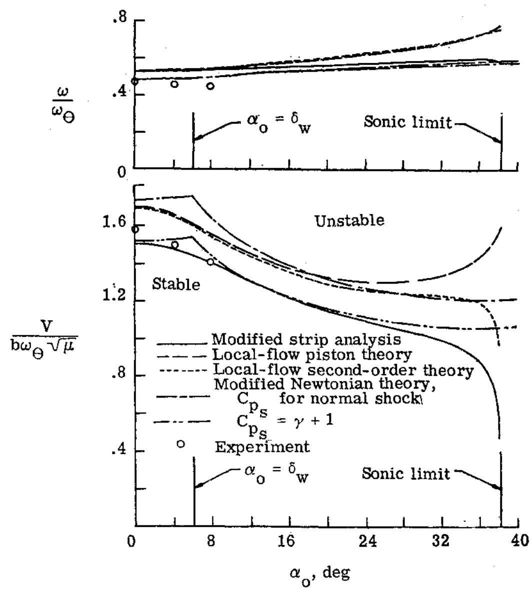
Comparisons of Calculated and Measured Flutter Characteristics for $M = 10$

Flutter characteristics for the wings of Ref. 1 at Mach number 10 have been calculated by use of the aerodynamic theories previously described and are compared with experimental data in Fig. 3. All of the methods show an extensive range of angle of attack below shock detachment angle in which the flutter speed is adversely affected by angle of attack.

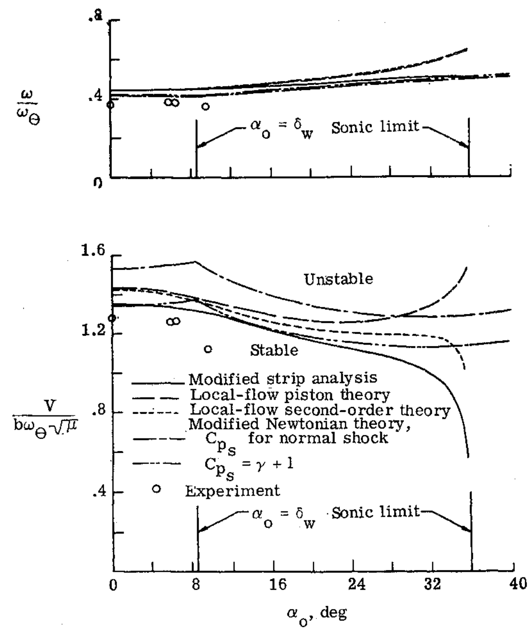
Values of flutter-speed index $V/b\omega_\theta(\mu)^{1/2}$ obtained from the modified strip analysis and from the modified Newtonian flow theory, incorporating $C_{ps} = \gamma + 1$, are both in reasonable agreement with the limited available experimental data. However, the modified Newtonian flow theory with $C_{ps} = \gamma + 1$ takes no account of variations in Mach number and hence is likely to be useful only for hypersonic speeds. It is noted, however, that some effect of variations in Mach number might be incorporated in a manner similar to that shown in Ref. 31, that is, by using $C_{ps} = (\gamma + 1)M^2/\beta^2$ instead of $C_{ps} = \gamma + 1$.

In comparison, both local flow methods and the Newtonian flow theory incorporating a value of C_{ps} associated with a normal shock do account in some measure for variations in Mach number but give flutter results that are unconservative (higher flutter speed index) relative to experiment for both wings. For relatively small values of α_0 and δ_w, the use of normal shock values of C_{ps} would seem less appropriate than use of the small disturbance value $C_{ps} = \gamma + 1$. At high angles, on the other hand (e.g., above shock detachment angle) (Figs. 3 and 4), the normal shock value may be the more logical choice. Confirming experimental data, however, are lacking.

For mean angles of attack α_0 less than the wedge half angle δ_w, the Newtonian theory predicts flutter speeds rising slightly with increasing α_0—a trend which is not borne out by the



a) $t/c = 0.1062$.



b) $t/c = 0.1505$

Fig. 3 Comparison of flutter characteristics at $M = 10$ as obtained from several methods ($g = 0.02$).

experimental data. An abrupt slope change occurs in the calculated curves at $\alpha_0 = \delta_w$ because below that angle of attack only the two forward surfaces of the diamond airfoil are "exposed" to the flow, whereas above that angle, only the two lower surfaces are "exposed." The variation of aerodynamic loading (and hence, flutter characteristics) with mean angle of attack is, of course, different for these two conditions.

At least some of the differences between calculation and experiment shown in Fig. 3 are attributed to the effects of finite span (neglected only for simplicity in the present calculations) and boundary layer (see Appendix). Both of these effects should be destabilizing[32,33] and both should be more prominent for the thicker wing (wing 1505).

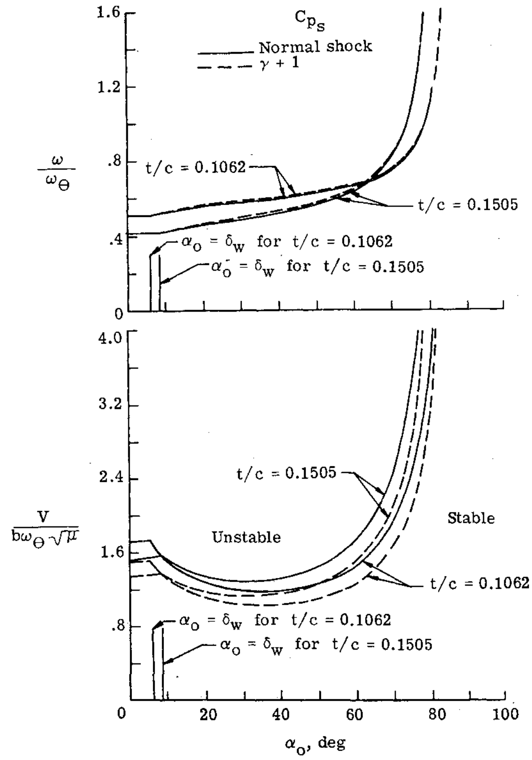


Fig. 4 Flutter characteristics obtained from modified Newtonian theory at $M = 10$ ($g = 0.02$).

Flutter Near Shock Detachment

The results of the modified strip analysis show a sharp drop in flutter speed as shock detachment angle is approached. In contrast, the corresponding flutter frequencies (Fig. 3) are relatively insensitive to variations in angle of attack. The steepness of the decline in flutter speed index results from the concurrent steep rise in C_{N_α} and rapid forward movement of the aerodynamic center as obtained from shock expansion theory (Fig. 2). Both of these effects are usually destabilizing.

The results from local flow second-order theory show a similar decline in flutter speed near detachment; however, the results from local flow piston theory rise as detachment is approached. This difference in trend results from the previously mentioned appearance of a factor $1/M_L$ in the expression for lifting pressure as obtained from local flow piston theory and a corresponding factor $1/\beta_L$ in the expression obtained from local flow, second-order theory. At the sonic limit, M_L becomes 1 and β_L becomes 0 on the lower forward surface of the airfoil. Hence, for that condition, local flow piston theory gives finite lifting pressures, whereas local flow, second-order theory gives infinite lifting pressures and vanishing flutter speed. Both of these local flow methods predict flutter speeds unconservatively in the experimental range of α_0, and neither agrees with experiment as well as the modified strip analysis, so the present limited results offer little to recommend these local flow methods.

The calculated flutter speeds given by Newtonian theory vary smoothly (Fig. 3) through detachment as Newtonian theory does not account for the occurrence of shock detachment. As the forward lower surface approaches the vertical, the flutter speed index rises sharply (Fig. 4).

The authors have not previously seen flutter calculations for conditions near shock detachment by any method that took into account the detachment phenomena. The present results, however, indicate not only the possibility of important deterioration in flutter margins as angle of attack increases from zero but also the possibility of a more drastic decline near detachment. Although no experimental data appear to

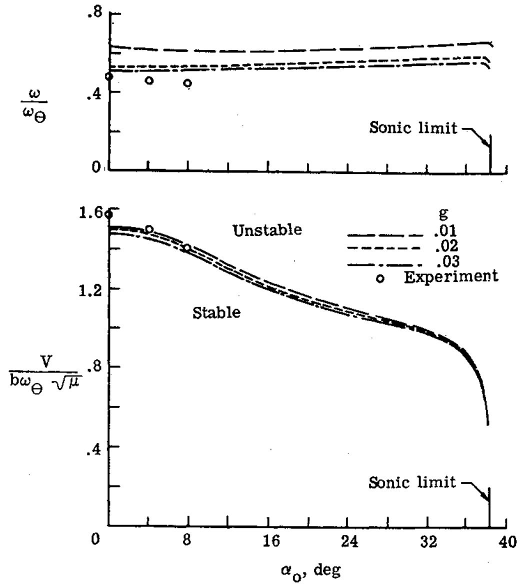
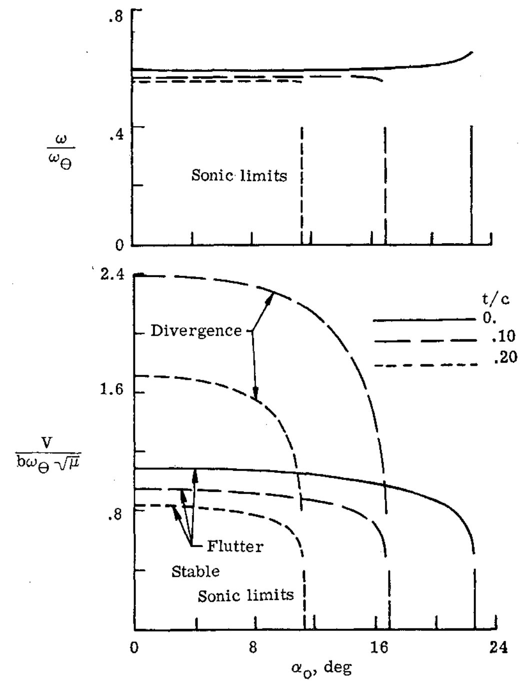


Fig. 5 Effect of structural damping on flutter characteristics of wing 1062 at $M = 10$.

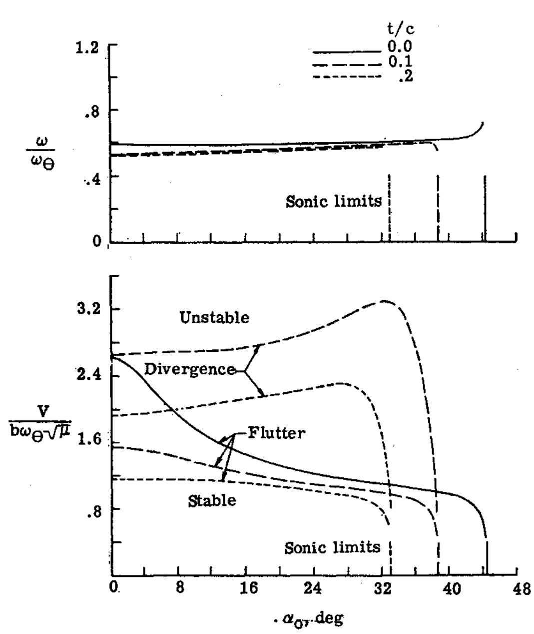
exist to confirm the latter behavior, the present results should at least be regarded as cautionary.

For $\alpha_0 = 0$, experimental flutter data that encompass the shock detachment Mach number are, of course, available. Most of these results, however, are for wings with blunt leading edges, for which there is no shock detachment condition in the classical sense, or for wings with finite span and sweepback, both of which are expected to ameliorate the sharpness and depth of a dip in flutter speed index associated with detachment. Furthermore, many of the available experimental flutter results do not contain enough flutter points in the immediate vicinity of detachment to indicate clearly whether the sharp drop in flutter speed predicted herein actually occurs. One notable exception may be Ref. 34, in which some of the flutter speed data for a rectangular wing with hexagonal airfoil show an abrupt and steep rise as the detachment Mach number of about 1.16 is exceeded. At this low Mach number, however, shock waves are relatively weak and any dip that actually occurs will probably be less pronounced than if detachment occurred at a higher Mach number. In addition, as previously mentioned, the influence of finite span should also serve to reduce the sharpness and depth of such a dip.

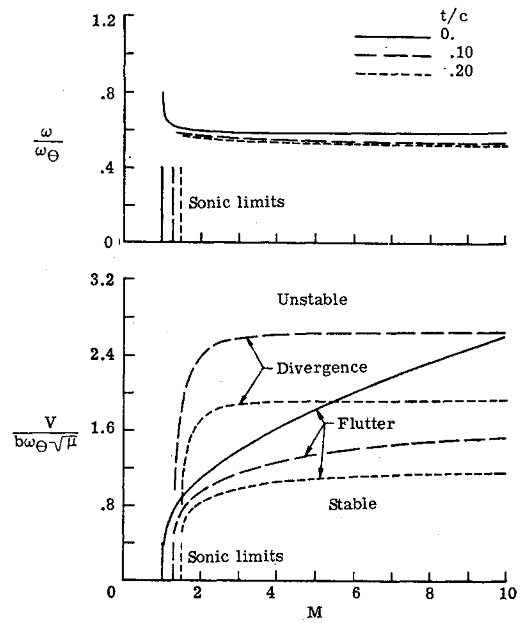
It seems unlikely that an abrupt phenomenon such as shock detachment would have no effect on flutter, as is predicted by Newtonian theory, nor as drastic an effect as is predicted by the modified strip analysis with aerodynamic parameters from shock expansion theory. Nevertheless, some type of dip may be anticipated. Furthermore, the flowfield for the high Mach number, high angle-of-attack condition involves strong shock waves bounding an embedded transonic flow and may have a rather different effect than the usual detachment condition at



b) Variation with angle of attack at $M = 2$.



a) Variation with angle of attack at $M = 10$.



c) Variation with Mach number at $\alpha_0 = 0$.

Fig. 6 Effect of thickness on flutter and divergence characteristics of wing 1062 ($g = 0.02$).

$\alpha_0 = 0$. Thus, the possibility exists that the vicinity of shock detachment may be a critical region for supersonic vehicles that operate at high angles of attack.

Parametric Study

The effects on flutter characteristics of variations in structural damping g, wing thickness ratio t/c, center-of-gravity position x_θ, and modal frequency ratio ω_h/ω_θ have been examined by the use of the modified strip analysis.

Structural damping coefficient

The influence on calculated flutter characteristics of variations in structural damping coefficient g is shown in Fig. 5. The results are seen to be relatively insensitive to changes in g in the vicinity of $g = 0.02$, the value estimated in Ref. 1 and used in the balance of the present calculations.

Effects of wing thickness

The destabilizing effect on flutter and divergence of increasing wing thickness (Fig. 6), which is familiar for $\alpha_0 = 0$ (e.g., Fig. 6c and Refs. 17–19 and 35) is shown to persist throughout the angles of attack and Mach numbers covered. However, at $M = 10$ the degradation of flutter speed index as α_0 increases from zero is indicated to be most rapid for the thinnest wings. For $M = 2$, on the other hand, the effect of increasing α_0 from zero is initially insignificant for all three wing thicknesses, and the deterioration of flutter speed index is mainly confined to the approach to shock detachment. It should be remembered, however, that the detachment angle of attack decreases as Mach number decreases.

Thus, it appears that for flutter clearance at the higher supersonic Mach numbers, the effects of even small angles of attack should be examined, whereas in the lower supersonic range, angle of attack may reasonably be neglected until it becomes an appreciable fraction of the detachment angle. In all cases, however, the vicinity of shock detachment should be investigated if that condition lies within or near the operating envelope of the vehicle.

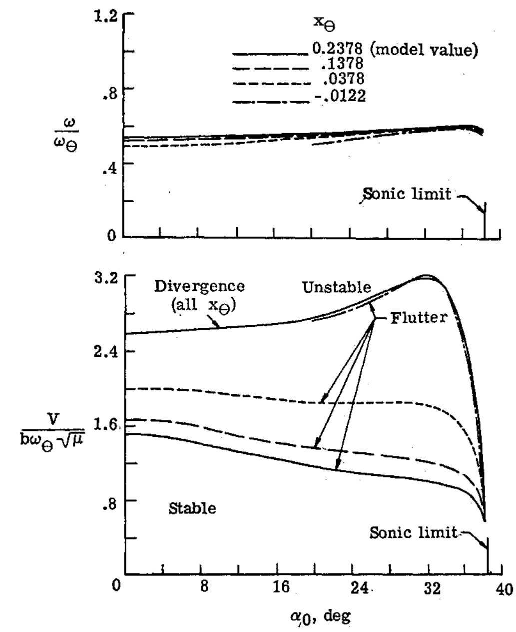
Effects of center of gravity

For the present wings at $M = 10$ and angles of attack well below detachment, the aerodynamic center moves aft with increasing α_0, while the normal-force slope C_{N_α} is increasing (Fig. 2a). These are usually opposing influences with regard to flutter behavior (stabilizing and destabilizing, respectively). Since the distance between wing-section center of gravity and section aerodynamic center is important in determining sensitivity of flutter speed to variations in aerodynamic center,[36,37] a parametric variation in center-of-gravity location x_θ (fraction of semichord aft of pitch axis which is at midchord for the present wings) was conducted for $M = 10$ in order to determine whether the effect of increasing angle of attack could be made negligible or even beneficial over some appreciable range of α_0.

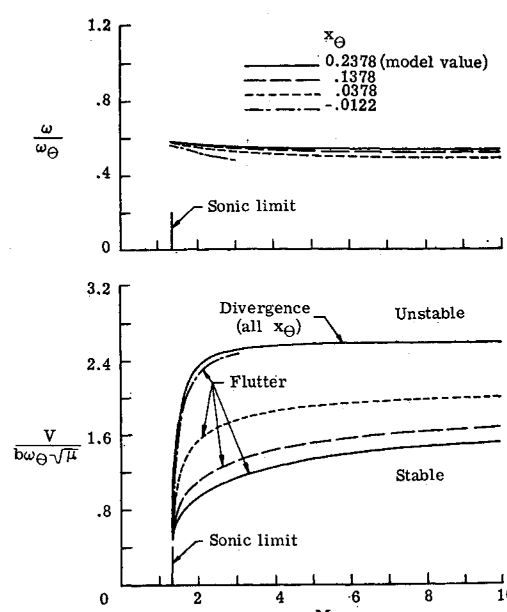
The results (Fig. 7a) show that the beneficial effect of moving center of gravity forward, which is well known for $\alpha_0 = 0$ (see also Fig. 7b), also occurs for the higher angles. Furthermore, by moving the center of gravity forward, it is indeed possible to offset the previously indicated detrimental effects of increasing α_0 up to the characteristic plunge as shock detachment is approached. Moving the center of gravity forward by one-tenth chord from the model values of Ref. 1 (x_θ from 0.2378 to 0.0378 for wing 1062) reduces flutter speed variations to less than 10% for both wings up to $\alpha_0 = 32°$. A further forward movement of 2.5% chord (to $x_\theta = -0.0122$ for wing 1062) results in no flutter at the lower angles followed by a region of rising flutter speed up to the inception of the characteristic plunge as shock detachment is approached.

Divergence speeds, on the other hand, remain finite at all angles of attack up to the sonic limit for all values of x_θ.

Thus, forward location of the wing section center of gravity, a familiar means of improving flutter speeds at $\alpha_0 = 0$, is predicted to alleviate the adverse effects of increasing angle of attack at supersonic and hypersonic speeds, at least up to the vicinity of shock detachment. Such alleviation may, however, depend strongly on airfoil shape. For example, the aerodynamic center for a wedge airfoil does not experience the favorable rearward shift with increasing α_0 that was indicated previously for the present diamond sections. Hence, the degradation of flutter speed with increasing angle of attack would probably be more severe for a wedge of comparable thickness than for the diamond sections examined herein. Furthermore, it is unlikely that the degradation for the wedge could be offset to any great extent by relocation of the section center of gravity.



a) Variation with angle of attack at $M = 10$.



b) Variation with Mach number at $\alpha_0 = 0$.

Fig. 7 Effect of center of gravity on flutter and divergence characteristics of wing 1062 ($g = 0.02$).

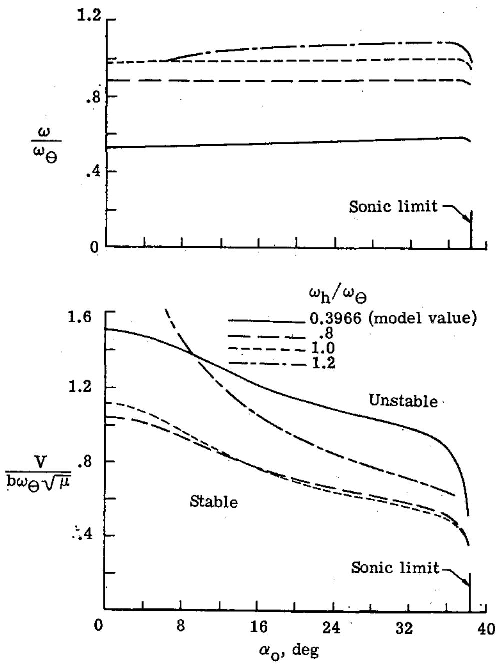


Fig. 8 Effect of modal frequency ratio on flutter characteristics of wing 1062 at $M = 10$ ($g = 0.02$).

Effects of modal frequency ratio

It is well known[17] that supersonic-hypersonic flutter speeds may be significantly affected by variations in modal frequency ratio ω_h/ω_θ and that minimum values of flutter speed usually occur near $\omega_h/\omega_\theta = 1$. In order to examine the extent to which this behavior persists as angle of attack increases, the modal frequency ratio has been parametrically varied for wing 1062. The results (Fig. 8) show that this behavior does indeed persist for angles of attack up to near detachment, and that the minimum flutter speeds occur for values of ω_h/ω_θ in the vicinity of 0.8 to 1.0. Figure 8 also indicates that the adverse effects of increasing angle of attack may be more pronounced for $\omega_h/\omega_\theta > 1.0$ (a range pertinent to some all-moving controls) than for $\omega_h/\omega_\theta < 1.0$. For $\omega_h/\omega_\theta = 1.2$, the present calculations indicate no flutter below about 6° angle of attack at $M = 10$; however, flutter is predicted to occur even at zero angle of attack for Mach numbers below about 6 (Fig. 10b of Ref. 30).

Conclusions

This exploratory analytical investigation has revealed several features of the methods investigated and of supersonic-hypersonic flutter at angle of attack. The results of the modified strip analysis and the modified Newtonian theory with $C_{p_s} = \gamma + 1$ were in reasonable agreement with the limited experimental flutter data which were available only for angles of attack up to about 10°. The local flow piston and second-order theories, and modified Newtonian theory with C_{p_s} for a normal shock produced speeds that were unconservative relative to experiment. Further, the modified strip analysis is indicated to be a unified, yet simple technique for flutter analysis over a wide range of supersonic-hypersonic Mach numbers, angles of attack, and airfoil shapes.

The flutter speeds were generally affected adversely by increasing angle of attack, and the vicinity of shock detachment, a phenomenon not accounted for by Newtonian theory, is indicated to be a potentially critical flutter region which warrants further attention. A brief parametric study indicated that the detrimental effect on flutter speed of increasing angle of attack from zero is most pronounced for the thinnest airfoils. Forward location of center of gravity is shown, however, to have an alleviating effect on this degradation.

Appendix

Comments on Treatment of Finite Wings

As indicated previously, the assumption of two-dimensionality in the present analysis is not essential but is made for convenience only. An attempt at a more accurate representation of the finite span wings of Ref. 1 would incorporate a) the effects of at least the lower vibration modes involving structural deformation, and b) the three dimensional aerodynamic effects of finite span as in the more general form of the modified strip analysis.[5–8,13] Although piston theory and Newtonian flow theory are based on point function pressure expressions and hence take no explicit account of finite planforms, these methods may also be modified in a straightforward manner to incorporate the approximate effects of finite span.[32]

In addition, the effects of boundary layer could be approximated (if separation is not involved), for example, by modifying the airfoil shape by adding the steady-state, boundary-layer displacement thickness to the local wing thickness. Such quasi-steady modifications are justified by the very low values of reduced frequency that are typical of supersonic-hypersonic flutter. For wings of the present type, the primary influence of boundary layer, at least at the lower angles of attack, is probably to increase the effective leading-edge wedge angle and to permit some pressure carryover past the expansion corner of the airfoil.[38]

Aerodynamic effects of finite span and boundary layer vary, of course, with both angle of attack α_0 and Mach number M. Specifically, the finite tip becomes more important as α_0 increases and as M decreases.

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